with the traveled distance which is over 10^8 n miles in 235 days. The drift distance can be positive or negative, depending on the ballistic coefficient. If a satellite ballistic coefficient is smaller than that of the reference satellite β_1, the circumferential drift distance will be positive and vice versa.

The effects of $\Delta\beta$ on drift altitude are shown in Fig. 4 for various initial and final altitudes. For the conditions given in the example in the previous paragraph, the corresponding drift altitude is about $\Delta h \cong 0.0065$ n miles.

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Forces Due to Gaseous Slot Jet Boundary-Layer Interaction

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Nomenclature

d_j = jet slot width
F = normal force per unit width
I = specific impulse based on jet mass flow
K = Crocco-Lees profile parameter
k = mixing-rate coefficient
L = distance from leading edge to jet
M = Mach number
$\dot{m}$ = mass flow rate
P = pressure
R_e = Reynolds number
u = longitudinal velocity
x = distance along plate
y = distance normal to plate
γ = ratio of specific heats
ρ = density

Subscripts

j = jet exit
$0j$ = jet stagnation
P = plateau
∞ = freestream

Introduction

THE use of reaction jets for control of high-altitude aerodynamic vehicles or space vehicles is well known. It is also of interest to consider using this control method at all altitudes. The advantages are 1) replacement of structural control surfaces eliminating a possible aerodynamic heating problem and offering the possibility of a weight saving, and 2) omission of system overlap for a vehicle with wide altitude range. The main question, however, refers to the effective control forces. This note offers an approximate solution to one portion of this problem, i.e., the control forces due to the interaction between a supersonic boundary layer and a slot-type (i.e., two-dimensional) gaseous jet issuing normal to the vehicle surface. The results are presented as a ratio of effective aerodynamic control force to jet-reaction force. Comparisons are made with experimental data.

Analysis

The flow model under consideration is shown in Fig. 1, and the following assumptions are made:

1) The upstream boundary layer is either all laminar or all turbulent.
2) Separation has occurred upstream of the jet.
3) The majority of the upstream separated flow (i.e., after separation and before recompression) can be characterized as a constant pressure mixing region.
4) The jet Mach number is $\geqq 1.0$, and the jet profiles are slug type.
5) The region downstream of the jet is sufficiently short so that the net effective side force due to overexpansion is small and no mixing takes place.
6) The entire flow is isoenergetic.

Then, assuming a negligibly thin boundary layer at (0, 0′), a momentum balance over the volume (0, 0′, 2, 3, 4, 1, 0) yields†

$$\dot{m}_{34}u_{34} - \Delta\dot{m}_p u_p = P_p y_{12} - P_{34}y_{34} \tag{1}$$

The quantity $\Delta\dot{m}_p$ represents the mass influx through plane (0′-2). And, as a first approximation, $\Delta\dot{m}_p u_p$ is assumed to be totally imposed on the jet. The parameter $\Delta\dot{m}_p$ can be determined by approximating the Crocco-Lees mixing-rate coefficient[1] by

$$k = (\Delta\dot{m}_p/\Delta x_s)/\rho_p u_p \tag{2}$$

Then, assuming an average (k) over the mixing region, it is seen that the length of the mixing region (Δx_s) is given by

$$\Delta x_s = [P_{34}y_{34}(1 + \gamma_j M_{34}^2) - P_p y_{12}]/\gamma_\infty P_p M_p^2 k \tag{3}$$

For the downstream regions, mass conservation of the jet flow can be written as

$$P_{0j}d_j M_j\{1 + [(\gamma - 1)/2]M_j^2\} \exp[(1 + \gamma_j)/2(1 - \gamma_j)] = P_{34}y_{34}M_{34}\{1 + [(\gamma_j - 1)/2]\,M_{34}^2\}^{1/2} \tag{4}$$

Substituting (4) into (3)

$$\Delta x_s/d_j = (P_{0j}M_j\{1 + [(\gamma - 1)/2]M_j^2\} \exp\,[(1 + \gamma_j)/2(1 - \gamma_j)]\tilde{B})/\gamma_\infty P_p M_p^2 k \tag{5}$$

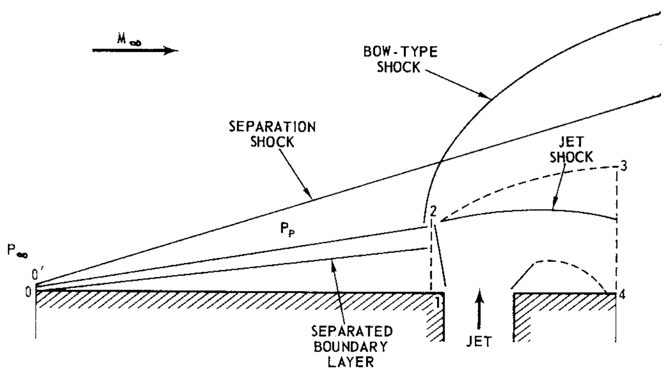

Fig. 1 Jet interaction model.

† Where the force on the downstream jet contact surface and the momentum due to jet curvature are assumed to be equal and mutually balancing.

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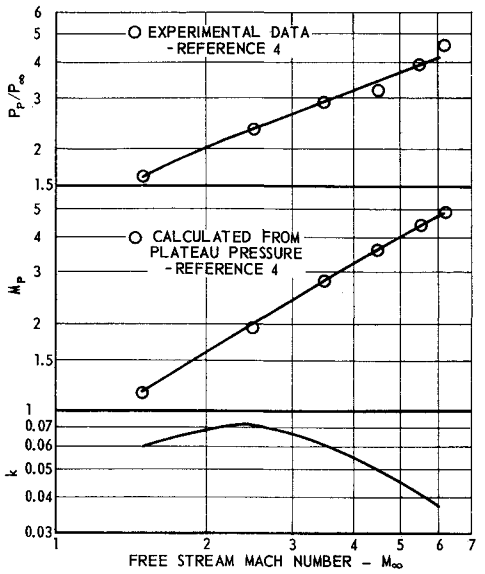

Fig. 2 Variation of the turbulent flow parameters (plateau pressure ratio, plateau Mach number, and mixing rate coefficient) with freestream Mach number.

where

$$\bar{B} = (1 + \gamma_j M_{34}^2)(1 - (y_{12}/y_{24})(P_p/P_{34}) \times \{1/[1 + \gamma_j M_{34}^2]\})/M_{34}\{(1 + [\gamma_j - 1)/2]M_{34}^2\}^{1/2} \quad (6)$$

The conditions at plane (3,4) are by no means easy to determine. Thus, for example, some investigators have chosen to let $y_{12} = y_{34}$. While this may be reasonable for the jet-shock boundary (Fig. 1), it is not obviously true for the outer edge of jet itself. Similar difficulties are encountered in attempts to determine the pressure (P_{34}) and Mach number (M_{34}). The assumption here will be that the term involving y_{12}/y_{34}, in the brackets of Eq. (6), is small compared to 1.0 (see Appendix), so that $\bar{B}$ can be calculated solely from M_{34}. Then, since B is a weak function of M_{34}, the hypersonic simplification will be made giving

$$\bar{B} \cong \gamma_j/[(\gamma_j - 1)/2]^{1/2} \quad (7)$$

Now, define the effective side force due to the aerodynamic interaction as

$$F_w = (P_p - P_\infty)\Delta x_s \quad (8)$$

and the idealized jet-reaction force as

$$F_j = P_\infty d_j((P_{0j}/P_\infty)(1 + \gamma_j M_j^2) \times \{1 + [(\gamma_j - 1)/2]M_j^2\}^{\gamma_j/1-\gamma_j} - 1) \quad (9)$$

Substituting Eq. (5) into the ratio of Eqs. (8) to (9), we see

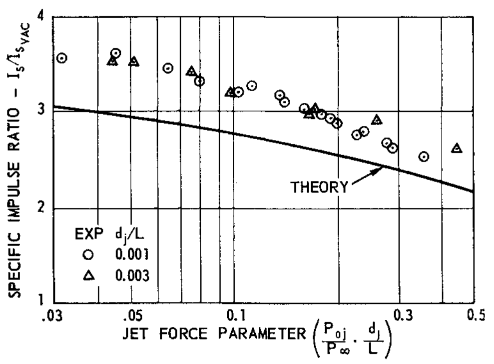

Fig. 3 Specific impulse ratio, laminar flow (experimental data from Ref. 2, M_∞ = 4).

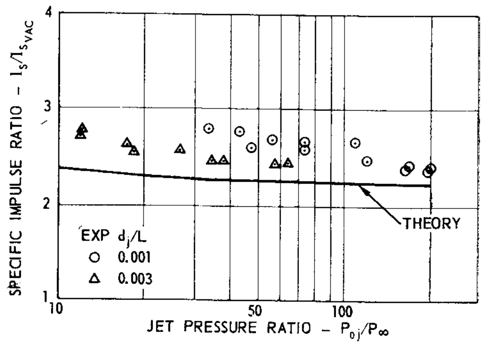

Fig. 4 Specific impulse ratio turbulent flow (data from Ref. 2, M_∞ = 4).

that

$$F_w/F_j = (1 - P_\infty/P_p)M_j[1 + [(\gamma_j - 1)/2]M_j^2] \times \exp[(1 + \gamma_j)/2(1 - \gamma_j)]\bar{B}/\{\gamma_\infty k M_p^2 (1 + \gamma_j M_j^2) \times [1 + [(\gamma_j - 1)/2]M_j^2]^{\gamma_j/(1-\gamma_j)} - P_\infty/P_{0j}\} \quad (10)$$

The remaining tasks are to define the mixing-rate coefficient (k), the plateau Mach number (M_p), and pressure (P_p).

For laminar flow, the empirical equation given by Amick and Carvalho[2] can be employed to determine plateau pressure

$$P_p/P_\infty = 1 + 1.27[(L/d_j)/(L/d_j - \Delta x_s/d_j)]^{1/4} M_\infty^{3/2} R_{e_{L\infty}}^{1/4} \quad (11)$$

where $R_{e_{L\infty}}$ is the Reynolds number based on the freestream conditions and the distance from the leading edge to the jet (L). The plateau Mach number can then be obtained from a simple isentropic compression. As a first approximation, the average value of k is taken as the separation value from the work of Glick[3] where

$$k \cong \bar{C}(1 - K_s)/(0.44\, R_{e_{x,p}})^{1/2} \quad (12)$$

Here, $R_{e_{x,p}}$ is the average Reynolds number of the separated flow based on the distance to separation ($L - \Delta x_s$), and K_s is the Crocco-Lees profile parameter at separation (taken equal to 0.63 by Glick). The constant $\bar{C}$ is found to be approximately 15 by Glick.

For turbulent flow, the empirical plateau pressure correlation of Sterrett and Emery[4] has been used to calculate plateau Mach numbers (see Fig. 2). The mixing-rate coefficient variation throughout the separated region is not well known. As a first approximation, the value of k at separation is taken from previous theoretical work by the present author[5] and employed for the calculations shown here. Turbulent mixing-rate coefficients are shown as a function of freestream Mach number in Fig. 2.

Sample Calculations

Comparisons between theory and experiment have been made using the data of Amick and Carvalho.[2] Figure 3 shows the calculations for laminar flow, and Fig. 4 presents the turbulent calculations. Agreement is seen to be favor-

Table 1 Order of magnitude calculation for jet turning region

	$(y_{12}/y_{34})(P_p/P_{34})/(1 + \gamma_j M_{34}^2)$		
	M_∞		
P_{0j}/P_∞	2	4	6
10	0.047	0.156	0.302
100	0.013	0.045	0.086
1000	0.005	0.016	0.032

able in both cases.‡ It is interesting to note that both the laminar theory and experiment show that at a particular Mach and Reynolds number the proper jet parameter for nondimensionalization is $(P_{0j}/P_\infty) \times (d_j/L)$. The turbulent theory indicates that P_{0j}/P_∞ is a sufficient jet parameter. The data appears to confirm this. In both cases the theory is seen to fall on the conservative side.

Conclusions

An approximate theoretical method for calculating side forces due to slot jet-boundary layer interaction has been presented. The results of sample calculations indicate that the theory predicts the correct qualitative trends and is quantitatively conservative. Finally, and most significantly, the importance of viscous mixing to the jet force interaction is brought out.

Appendix

Estimates of the bracketed term involving y_{12}/y_{34} in Eq. (6) can be made based on the experimental data of Refs. 6 and 7. Zukoski and Spaid[6] derive a relation [their Eq. (3)] for a penetration height which roughly corresponds to, and is somewhat less than, what is termed here y_{34}. Strike et al.[7] set down an empirical relation for the penetration distance of the jet shock [their empirical Eq. (1)] which roughly corresponds to y_{12}. Although these data pertain to jet injection through circular holes in plates with long downstream runs, rather than the flap-type slots under consideration here, the information is useful for estimation purposes. Table 1 contains calculations of $(y_{12}/y_{34})(P_p/P_{34})/(1 + \gamma_j M_{34}{}^2)$ based on expansion of the jet to P_∞, $\gamma_j = 1.4$ and use of the foregoing equations for y_{12} and y_{34}. (Turbulent flow has been assumed as this will be the more critical case.) These calculations indicate that, except for the combination of low jet pressure ratio and high Mach number (where the model applicability is questionable anyway), the assumption of $(y_{12}/y_{34})(P_p/P_{34})/(1 + \gamma_j M_{34}{}^2) \ll 1$ appears to be valid.

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‡ Since the submittal of this note, additional experimental data have been graciously made available by N. E. Hawk, University of Michigan. These data were taken employing side plates on a two-dimensional model and are in even better agreement with the theoretical predictions of this note than are the data shown here.

Transient Performance Prediction of a Nonadiabatic, Real-Gas Propulsion System

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Nomenclature

C_v	=	heat capacity at constant volume
E	=	internal energy
E_c	=	$pv\dot{m}/J$, compression energy
E_t	=	energy difference between inlet slug and tank gas
g	=	conversion constant
H	=	enthalpy
I_{ideal}	=	impulse
$I_{\text{sp}_{\text{ideal}}}$	=	specific impulse
J	=	conversion constant
M	=	mass
$\dot{m}$	=	mass flow of nozzle
m	=	mass increment
P	=	pressure
P_e	=	nozzle exit pressure
P_a	=	atmospheric pressure
Q	=	heat flow
S	=	entropy
T	=	temperature
v	=	specific volume
W	=	weight flow rate
α	=	divergence half-angle of nozzle
ϵ	=	nozzle area ratio

THE use of a gas as a propellant in a simple gas-propulsion system or as a pressurant may require the detailed analysis of the thermodynamic processes occurring within the system. The nature of most gases and the high pressures immediately preclude the use of the simplified equations representing ideal gases, and the engineer must resort to extensive tables, charts, or formidable, empirical equations of state. The analysis of the processes occurring within a gaseous system requires the determination of the state of the gas (p, v, T, H, and E), which is a difficult task for a nonideal gas that is being compressed or expanded by the addition or subtraction of gas. The difficulty is compounded when one must consider variable heat gains or losses and variable temperature mass input.

This paper presents a technique for predicting the state of any gas for which equations of state are available under the nonadiabatic transient conditions of loading (compression) or use (expansion) from a storage container. The method uses the basic differential equations of thermodynamics relating the change in internal energy, enthalpy, and entropy to the independent variables. The equations, which are put in finite-difference form, are completely general, and thus the method does not require any assumptions regarding ideality of gas. The solutions are obtained via a digital computer program, which solves the thermodynamic equations incrementally, utilizing the equations of state, the work done, the heat loss, and the mass change for inputs at each point. Since a relationship can be obtained for a nozzle for the specific impulse as a function of the state of the gas (e.g., enthalpy vs I_{sp}), then the total available impulse at any time for a system can also be determined. The validity of the method is tested by comparison of predicted temperatures and pressures to the actual values obtained during the filling and expansion of a tank with "Freon 14," a gas that is far from ideal.

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